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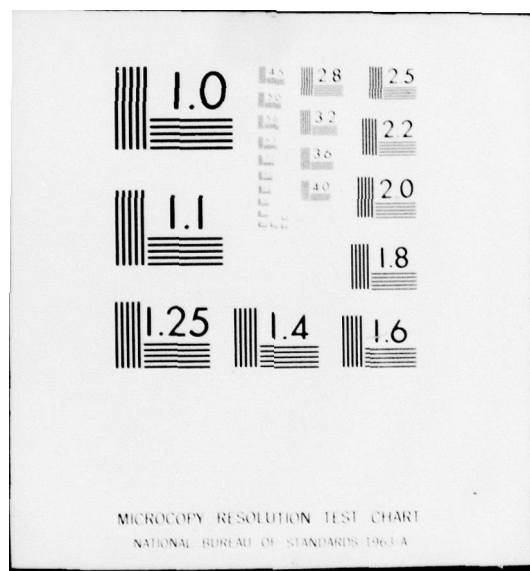
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## FOREIGN TECHNOLOGY DIVISION

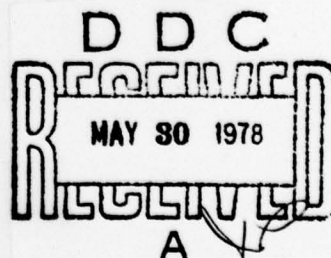


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ON THE THEOREM OF N. V. SMIRNOV WITH RESPECT  
TO A COMPARISON OF TWO SAMPLINGS

by

I. D. Kvit



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## EDITED TRANSLATION

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# U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<b>А а</b>	A, a	Р р	<b>Р р</b>	R, r
Б б	<b>Б б</b>	B, b	С с	<b>С с</b>	S, s
В в	<b>В в</b>	V, v	Т т	<b>Т т</b>	T, t
Г г	<b>Г г</b>	G, g	У у	<b>У у</b>	U, u
Д д	<b>Д д</b>	D, d	Ф ф	<b>Ф ф</b>	F, f
Е е	<b>Е е</b>	Ye, ye; E, e*	Х х	<b>Х х</b>	Kh, kh
Ж ж	<b>Ж ж</b>	Zh, zh	Ц ц	<b>Ц ц</b>	Ts, ts
З з	<b>З з</b>	Z, z	Ч ч	<b>Ч ч</b>	Ch, ch
И и	<b>И и</b>	I, i	Ш ш	<b>Ш ш</b>	Sh, sh
Й й	<b>Й й</b>	Y, y	Щ щ	<b>Щ щ</b>	Shch, shch
К к	<b>К к</b>	K, k	Ъ ъ	<b>Ъ ъ</b>	"
Л л	<b>Л л</b>	L, l	Ы ы	<b>Ы ы</b>	Y, y
М м	<b>М м</b>	M, m	Ь ь	<b>Ь ь</b>	'
Н н	<b>Н н</b>	N, n	Э э	<b>Э э</b>	E, e
О о	<b>О о</b>	O, o	Ю ю	<b>Ю ю</b>	Yu, yu
П п	<b>П п</b>	P, p	Я я	<b>Я я</b>	Ya, ya

\*ye initially, after vowels, and after Ъ, Ь; e elsewhere.  
When written as ё in Russian, transliterate as yě or ě.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh <sup>-1</sup>
cos	cos	ch	cosh	arc ch	cosh <sup>-1</sup>
tg	tan	th	tanh	arc th	tanh <sup>-1</sup>
ctg	cot	cth	coth	arc cth	coth <sup>-1</sup>
sec	sec	sch	sech	arc sch	sech <sup>-1</sup>
cosec	csc	csch	csch	arc csch	csch <sup>-1</sup>

Russian English

rot curl  
lg log



ON THE THEOREM OF N.V. SMIRNOV  
WITH RESPECT TO A COMPARISON OF  
TWO SAMPLINGS

I.D. Kvit

(Presented by Academician A.N. Kolmogorov on 12 January 1950)

In work [1] N.V. Smirnov studied the following important problem of statistics: there are two series of results of independent observations on the random quantities  $\xi_1$  and  $\xi_2$

$$x_1, x_2, \dots, x_m$$

and

$$y_1, y_2, \dots, y_n.$$

It is asked under what conditions is it possible to consider that the distribution functions  $F_1(x) = P\{\xi_1 < x\}$  and  $F_2(x) = P\{\xi_2 < x\}$  are equal, and when is the divergence of the experimental data so considerable that the hypothesis  $F_1(x) = F_2(x)$  should be rejected.

In the present memorandum we show that the generalization of the theorems of A.N. Kolmogorov and N.V. Smirnov, obtained by G.M. Maniya [2] for one sampling, is transferred without difficulty to the problem of N.V. Smirnov about two samplings.

Let us examine the empirical distribution functions

$$S_m(x) = \frac{k_1(x)}{m},$$

where  $k_1(x)$  is the number of observed values of  $\xi_1$  less than  $x$  and

$$T_n(x) = \frac{k_2(x)}{n},$$

where  $k_2(x)$  is the number of observed values of  $\xi_2$  less than  $x$ .

At the discontinuity points we supplement the empirical distribution functions with the vertical segments.

In the indicated work N.V. Smirnov proved that if  $F_1(x) = F_2(x)$ , function  $F_1(x)$  is continuous and increases everywhere,  $\frac{m}{n} = \text{const}$ , then when

$$N = \frac{mn}{m+n} \rightarrow \infty$$

$$P\left\{D(m, n) \leq \frac{z}{\sqrt{N}}\right\} \rightarrow \sum_{k=-\infty}^{\infty} (-1)^k e^{-2k^2 z^2}, \quad (1)$$

where

$$D(m, n) = \sup_{-\infty < x < \infty} |S_m(x) - T_n(x)|.$$

Let us assume that  $\theta_1$  and  $\theta_2$  are arbitrary numbers,

$0 < \theta_1 < \theta_2 < 1$ . Let us assume that  $F_1(x) = F_2(x) = F(x)$  and determine  $\alpha$  and  $\beta$  by means of equalities

$$F(\alpha) = \theta_1, \quad F(\beta) = \theta_2.$$

Let us denote further

$$\alpha_{mn} = \min_x [S_m(x) = \theta_1; T_n(x) = \theta_1], \quad \beta_{mn} = \max_x [S_m(x) = \theta_2; T_n(x) = \theta_2].$$

Then when  $m \rightarrow \infty, n \rightarrow \infty$  in virtue of the theorem of Glivenko, there should be

$$\alpha_{mn} \rightarrow \alpha, \quad \beta_{mn} \rightarrow \beta.$$

Let us introduce the notations

$$D_{mn}^+(\theta_1, \theta_2) = \sup_{(\alpha_{mn}, \beta_{mn})} \{S_m(x) - T_n(x)\},$$

$$D_{mn}(\theta_1, \theta_2) = \sup_{(\alpha_{mn}, \beta_{mn})} |S_m(x) - T_n(x)|.$$

The obtained results can be formulated in the form of the following two theorems.

Theorem 1. If when  $n \rightarrow \infty$   $\theta_1^{(n)} = \theta_1 + o\left(\frac{1}{\sqrt{n}}\right)$  and  $\theta_2^{(n)} = \theta_2 + o\left(\frac{1}{\sqrt{n}}\right)$ ,  $0 < \theta_1 < \theta_2 < 1$  then

$$P \left\{ D_{mn}^+(\theta_1^{(n)}, \theta_2^{(n)}) \leq \frac{z}{\sqrt{N}} \right\} \xrightarrow{N \rightarrow \infty} \Phi^+(\theta_1, \theta_2; z)$$

where

$$\begin{aligned} \Phi^+(\theta_1, \theta_2; z) = & \frac{1}{2\pi \sqrt{1-R^2}} \int_{-\infty}^a \int_{-\infty}^b e^{-1/2 Q(z_1, z_2)} dz_1 dz_2 - \\ & - \frac{e^{-z^2}}{2\pi \sqrt{1-R^2}} \int_{-\infty}^{a'} \int_{-\infty}^{b'} e^{-1/2 Q(z_1, z_2)} dz_1 dz_2, \end{aligned}$$

$$a = \frac{z}{\sqrt{\theta_1(1-\theta_1)}}, \quad b = \frac{z}{\sqrt{\theta_2(1-\theta_2)}},$$

$$a' = \frac{z - 2z\theta_1}{\sqrt{\theta_1(1-\theta_1)}}, \quad b' = \frac{z - 2z(1-\theta_2)}{\sqrt{\theta_2(1-\theta_2)}}, \quad R = \sqrt{\frac{\theta_1(1-\theta_2)}{\theta_2(1-\theta_1)}},$$

$$Q(z_1, z_2) = \frac{1}{1-R^2} [z_1^2 + 2Rz_1z_2 + z_2^2],$$

$$\bar{Q}(z_1, z_2) = \frac{1}{1-R^2} [z_1^2 - 2Rz_1z_2 + z_2^2].$$

Hence, in particular,

$$P \left\{ D_{mn}^+(0, 1) \leq \frac{z}{\sqrt{N}} \right\} \xrightarrow{N \rightarrow \infty} 1 - e^{-z^2}.$$

We see thus that the theorem of N.V. Smirnov about the one-sided deviations of the empirical function from the theoretical is carried over to the case of the maximum of one-sided deviations of two empirical functions.

Theorem 2. In the suppositions of theorem 1

$$P \left\{ D_{mn}(\theta_1^{(n)}, \theta_2^{(n)}) \leq \frac{z}{\sqrt{N}} \right\} \xrightarrow{N \rightarrow \infty} \Phi(\theta_1, \theta_2; z),$$

where



$$\Phi(\theta_1, \theta_2; z) = \frac{1}{2\pi \sqrt{1-R^2}} \int_{-a}^a \int_{-b}^b e^{-1/2 Q(z_1, z_2)} dz_1 dz_2 -$$

$$- \frac{1}{\pi \sqrt{1-R^2}} \sum_{k=1}^{\infty} (-1)^{k-1} e^{-2k^2 z^2} \int_{-a_k}^{a_k} \int_{-b_k}^{b_k} e^{-1/2 Q(z_1, z_2)} dz_1 dz_2,$$

$$a_k = \frac{z - 2kz\theta_1}{\sqrt{\theta_1(1-\theta_1)}}, \quad b_k = \frac{z - 2kz(1-\theta_2)}{\sqrt{\theta_2(1-\theta_2)}}.$$

Hence when  $\theta_1=0, \theta_2=1$  we get (1).

Similar to the theorems of G.M. Maniya, the results discussed make it possible to use that interval of the observed values in which the results of the observations are more reliable.

In the work we used the method of Laplace transforms applied by W. Feller [3] for proofs of the theorems of A.N. Kolmogorov and N.V. Smirnov.

In conclusion I wish to express my deep thanks to Prof. B.V. Gnedenko for the statement of the problem and his guidance in the solving of it.

L'vov State University im. I. Franko      Submitted 12 January 1950

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- <sup>1</sup> Н. В. Смирнов, Бюлл. НИИМ МГУ, 2, в. 2 (1939).    <sup>2</sup> Г. М. Манья, ДАН, 69, № 4 (1949).    <sup>3</sup> W. Feller, Ann. of Math. Statistics, 19, 2, 177 (1948).